Investigation of Stochastic Heating in Low Collisionality Plasmas

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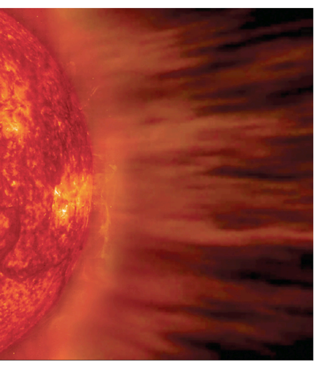
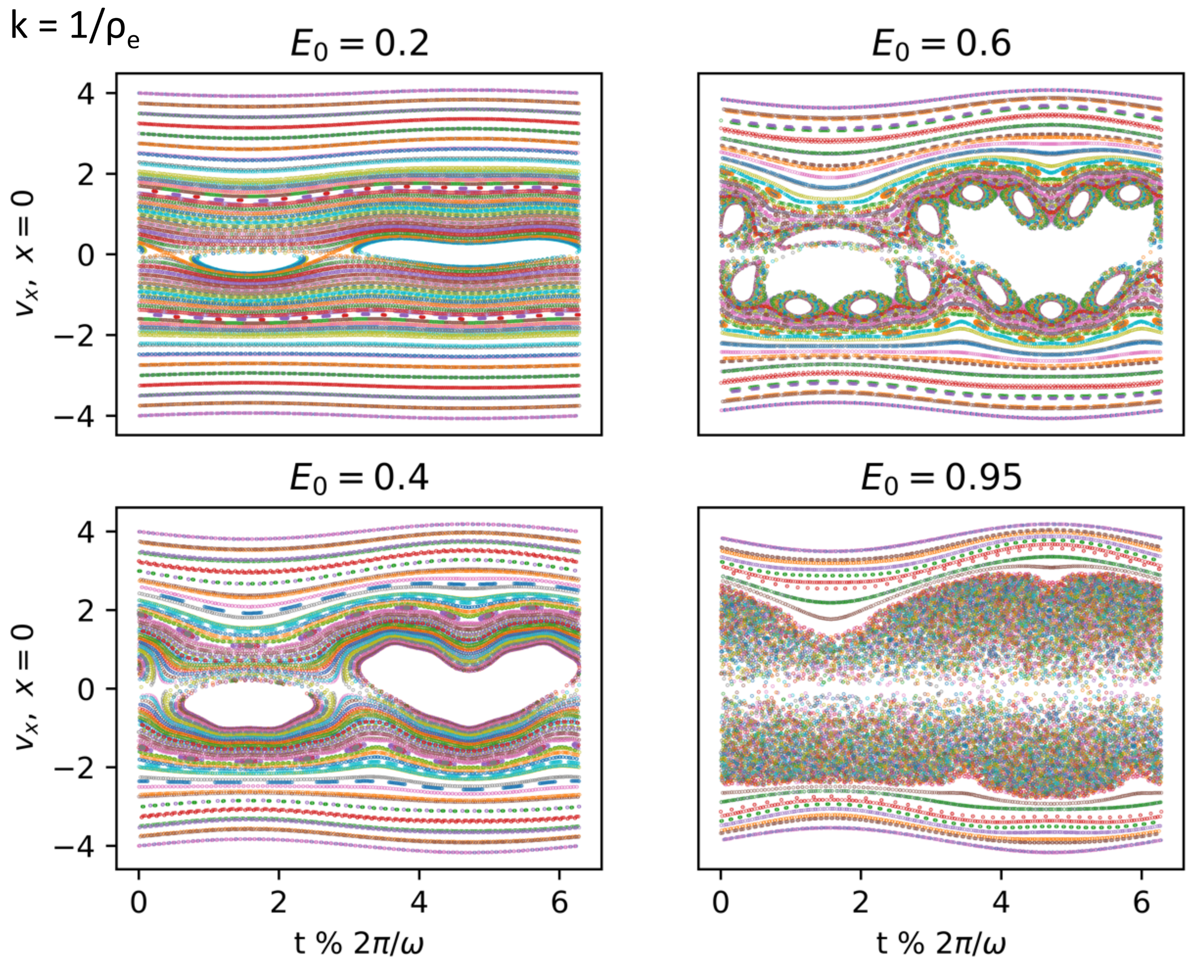
Signs of stochastic motion (chaos)



- The effect of stochastic heating in plasma is important in understanding the heating of the solar corona and the solar wind
- With the recent launch of the Parker Solar Probe Plus that will gather unprecedented data of the solar wind and corona, further theoretical investigation of stochastic heating processes is desired to compare with experimental data
- What does it take to drive particles into stochastic motion? A time and spatially varying **E** field perpendicular to a constant **B** field is sufficient

$$\mathbf{B} = B_0\mathbf{e}_z$$
$$\mathbf{E}(x,t) = E_0 \sin(kx - \omega t)\mathbf{e}_x$$

- This series Poincare plots at varying values of E_0 with $B_0 = 1$, $\omega = 0.4567$, and $k = 1/\rho_e$

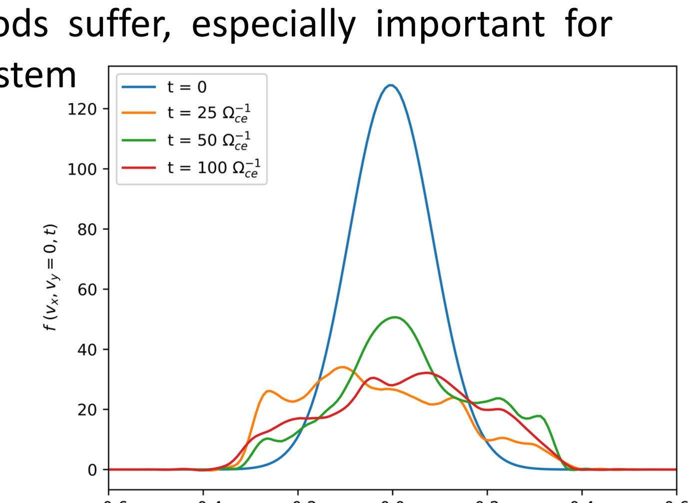


Continuum Vlasov-Maxwell solver

- The Vlasov-Maxwell system provides an accurate kinetic model for weakly collisional plasmas

$$\frac{\partial f_s}{\partial t} + \nabla_{\mathbf{z}} \cdot (\boldsymbol{\alpha}_s f_s) = C[f]$$
$$\nabla_{\mathbf{z}} = (\nabla, \nabla_{\mathbf{v}}), \quad \boldsymbol{\alpha}_s = \left(\mathbf{v}, \frac{q_s}{m_s}(\mathbf{E} + \mathbf{v}\times\mathbf{B})\right)$$
$$\frac{\partial \mathbf{B}}{\partial t} + \nabla\times\mathbf{E} = 0, \quad \nabla\cdot\mathbf{B} = 0$$
$$\epsilon_0\mu_0\frac{\partial \mathbf{E}}{\partial t} - \nabla\times\mathbf{B} = -\mu_0\mathbf{J}, \quad \nabla\cdot\mathbf{E} = \frac{\varrho_c}{\epsilon_0}$$
$$\varrho_c = \sum_s q_s \int_{-\infty}^{\infty} f_s\, d\mathbf{v}, \quad \mathbf{J} = \sum_s q_s \int_{-\infty}^{\infty} \mathbf{v} f_s\, d\mathbf{v}$$

- Directly solving a discretized Vlasov system avoids counting noise from which traditional particle-in-cell methods suffer, especially important for resolving stochasticity in a non-linear system
- Here a Maxwellian distribution function is evolved "passively," meaning evolving the Vlasov equation without Maxwell's equations that couple the plasma currents to the electromagnetic fields
- In the field configuration above for the highly stochastic regime, **flattening of the spatially integrated distribution is a signature of stochastic particle motions**



1D2V simulation, Nx = 48, Nvx = 48, Nvy = 48, polyOrder = 3

Collision Operator

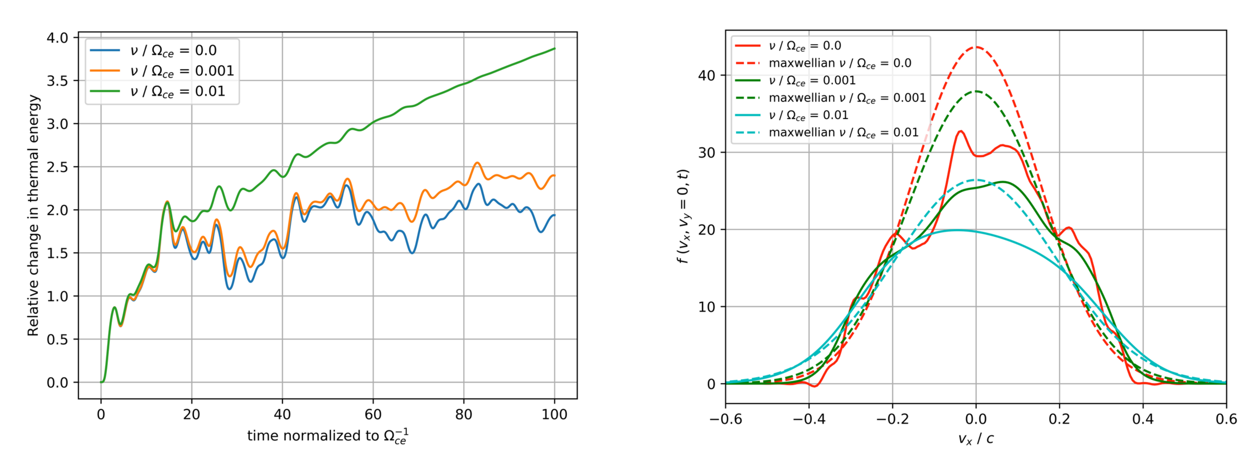
- To include collisions we use the Fokker-Plank collision operator, which is discretized according to the numerical DG scheme used in the Vlasov solver

$$\frac{\partial f}{\partial t} = \nu \frac{\partial}{\partial \mathbf{v}} \cdot \left[(\mathbf{v} - \mathbf{u}) f + v_t^2 \frac{\partial f}{\partial \mathbf{v}}\right]$$

- Inclusion of a collision operator provides **regularization of phase space structure** and an outlet for **energy dissipation**
- Collisions also create a physical source of **entropy increase** and hence makes the evolution of the distribution an **irreversible process**

Effect of collision operator on heating

- Driving fields produce stochastic motions in the plasma particles which then leads to an energy transfer into the plasma thermal energy
- Looking at this energy transfer in "passive" simulations with varying degrees of collisionality reveals how collisions can mediate stochastic heating



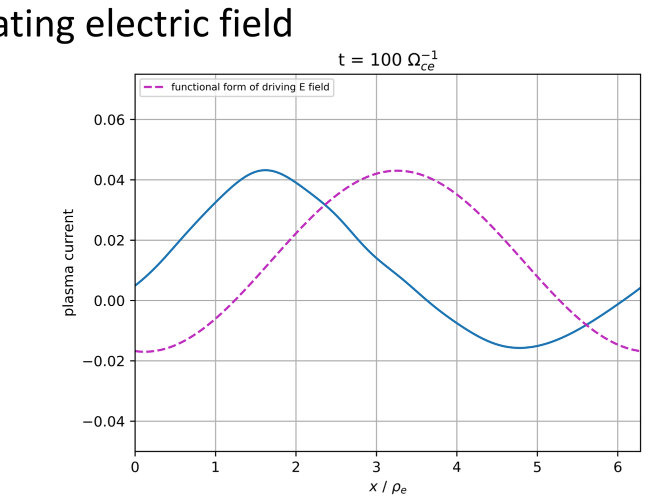
- In the collisionless case particles resonant with the electric field gain energy up until almost 20 cyclotron periods; after particles are driven off-resonance the thermal energy reaches a quasi-steady-state
- **Addition of collisions provides an outlet for further energy transfer from the electric waves to the particles**
- The transition from nonlinear resonant heating to collisional heating can be seen where the curves begin to diverge
- **Key takeaway:** with collisionality the thermal energy can increase without bound when particle currents do not feed back to fields
- Line-outs of the integrated 2D distribution function taken at $v_y = 0$ and $t = 100\ \Omega_{ce}^{-1}$ with superimposed Maxwellian distributions of the temperature indicated by the integrated thermal energy value at that time show the highly **non-Maxwellian nature of the distributions**
- Note that presence of phase space structure in the collisionless simulation is smoothed out when collisions are included

Current, entropy, and pressure in passive simulations

- In passive simulations the plasma sees only the user specified driving fields:

$$\mathbf{B} = B_0\mathbf{e}_z$$
$$\mathbf{E}(x,t) = E_0 \sin(kx - \omega t)\mathbf{e}_x$$

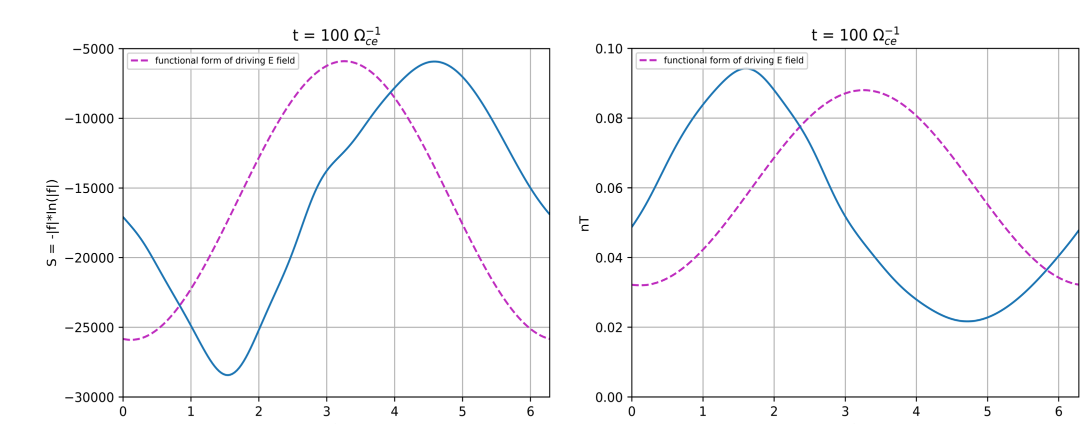
- This results in a clear correlation between the plasma current, numerical entropy, and pressure in relation the oscillating electric field



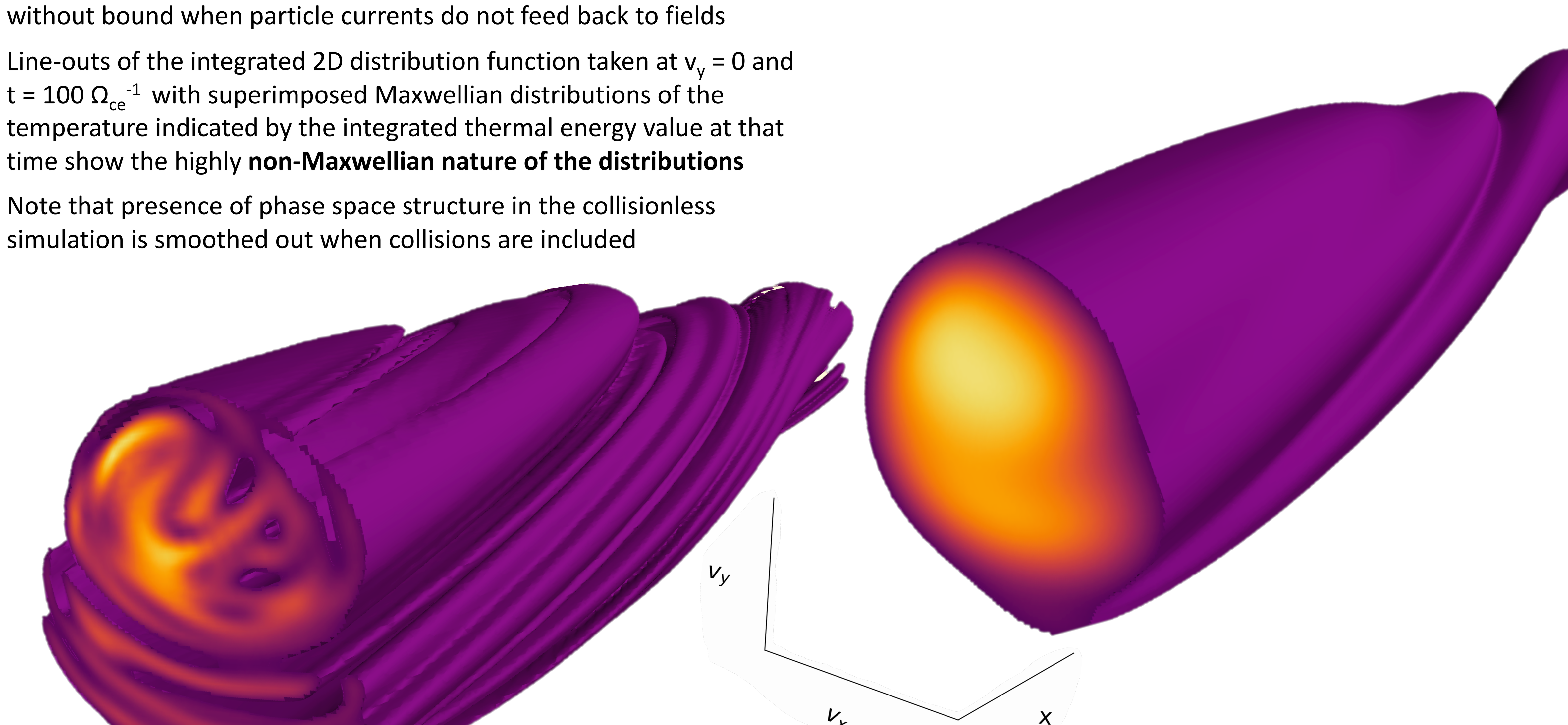
- Above shown is a π/2 phase shift between the plasma current and electric field in a collisional simulation well after the resonant heating period ends; a simple Fourier analysis of Ampere's law yields this expression for the current:

$$\mathbf{J}(x,t) = i\omega\epsilon_0 E_0 e^{ikx - i\omega t}$$

The factor of imaginary i mathematically represents a π/2 phase shift, affirming the simulation output



- A similar phase shift is seen with the numerical entropy and the plasma pressure

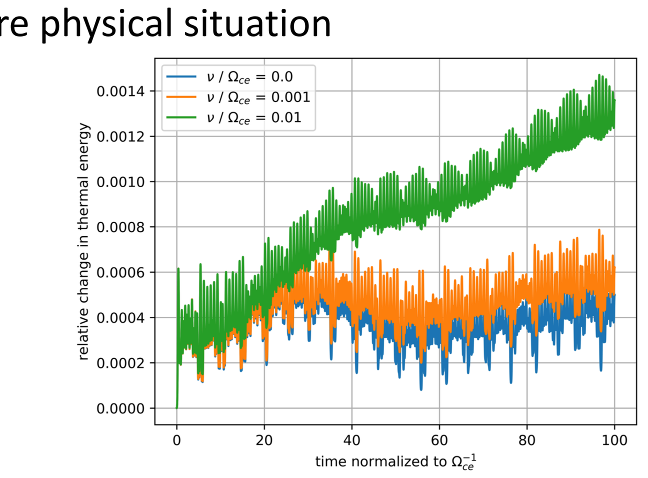


distribution functions at $t = 100\ \Omega_{ce}^{-1}$ (elongated axis is space, other axes are v_x and v_y)

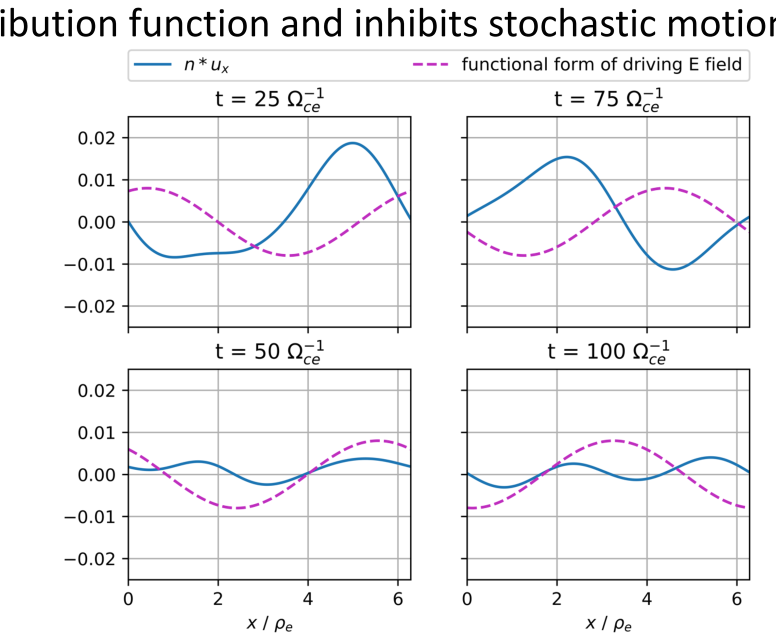
left: collisionless simulation; right collisional simulation $\nu / \Omega_{ci} = 0.01$

Self-consistent simulations

- Evolving the distribution function self-consistently such that the **particle currents feed back to the fields** by solving Maxwell's equations with the Vlasov equation provides a look at a more physical situation



- The same simulations ran before with self-consistency turned on leads to negligible increases in thermal energy
- **Rapid plasma response severely damps out the oscillating electric field** and leads to very minimal perturbation to the distribution function and inhibits stochastic motions



- **Key takeaway:** introduction of self-consistency leads to a plasma response that dramatically cancels the fields and prevents any stochasticity and subsequent heating
- A more complicated driver than a single electric wave mode may be required to see stochastic particle motions

Summary and future work

- Driving a magnetized plasma with a high amplitude perturbing electric field causes stochastic particle orbits and nonlinear resonances that lead to particle heating
- Collisions introduce a route for more electromagnetic field energy to dissipate into the plasma thermal energy
- Self-consistency leads to complete damping of driving field
- Future work will include "priming" the self-consistent simulation by initializing from a distribution that has the currents generated from the driving fields of a passive simulation; also investigating self-consistent simulations with delta- and time-correlated drivers

References and Acknowledgements

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[3] B.D.G. Chandran et al., Perpendicular Ion Heating by Low-frequency Alfvén-wave Turbulence in the Solar Wind, ApJ. 720 (2010) 503-515

- Code can be accessed
  - https://bitbucket.org/ammarhakim/gkyl/src/default/
  - conda install -c gkyl gkyl

- High performance computing was provided by Stampede2, Texas Advanced Computing Center (TACC) located at the University of Texas at Austin
- This project was supported by the DOE Summer Undergraduate Laboratory Internship program

Abstract: The effect of stochastic heating in plasma is important in understanding the heating of the solar corona and the solar wind. With the coming launch of the Parker Solar Probe Plus that will gather unprecedented data of the solar wind and corona, further theoretical investigation of stochastic heating processes is desired to compare with experimental data. In this study we use the continuum code Gkeyll to solve the Vlasov-Maxwell equations for the evolution of a plasma distribution function. Using a continuum solver avoids numerical issues such as particle noise characteristic of traditional particle-in-cell methods. We investigate the evolution of a distribution function in different electromagnetic (EM) field configurations such as those observed in solar wind conditions. In addition to test particle simulations, we will compare to distributions evolved self-consistently, such that the particle motion feeds back to the seed EM field, and including particle collision effects. Energy transfer from fields to particles will be studied to determine the extent to which stochastic motion of particles leads to particle heating.